

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions

M.M., Min.

Single choice Objective (no negative marking) Q.1,2,4	(3 marks, 3 min.)	[9, 9]
Subjective Questions (no negative marking) Q.3,5,6,7	(4 marks, 5 min.)	[16, 20]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. If $f(x) = \frac{x-1}{x+1}$, then $f(f(ax))$ in terms of $f(x)$ is equal to
 (A) $\frac{f(x)-1}{a(f(x)-1)}$ (B) $\frac{f(x)+1}{a(f(x)-1)}$ (C) $\frac{f(x)-1}{a(f(x)+1)}$ (D) $\frac{f(x)+1}{a(f(x)+1)}$
2. If $f(x) = ((\text{sgn } x)^{\text{sgn } x})^n$; n is an odd integer. Then
 (A) $f(x)$ is an odd function (B) $f(x)$ is an even function
 (C) $f(x) = 0$ (D) none of these
3. Let $f(x) = \frac{2}{4^x + 2}$ for real numbers x. Evaluate : $f\left(\frac{1}{2011}\right) + f\left(\frac{2}{2011}\right) + \dots + f\left(\frac{2010}{2011}\right).$
4. In which of the following functions, range is singleton set.
 (A) $f(x) = [x] + [-x]$ (B) $f(x) = \{x\} + \{-x\}$ (C) $f(x) = |\text{sgn}(x)|$ (D) $f(x) = [\sqrt{x - [x]}]$
 where $[x]$, $\{x\}$ and $\text{sgn}(x)$ are greatest integer function, fractional part function and signum function respectively.
5. Evaluate
 (i) $\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\cos \frac{x}{2} \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$ (ii) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{1+x \sin x} - \sqrt{\cos 2x}}{\tan^2(x/2)} \right)$
6. Solve the inequality : $(2 \log_3^2 x - 3 \log_3 x - 8) (2 \log_3^2 x - 3 \log_3 x - 6) \geq 3.$
7. Let $f(x) = \frac{|x^3 - 6x^2 + 11x - 6|}{x^3 - 6x^2 + 11x - 6}$. Find the set of points 'a' where $\lim_{x \rightarrow a} f(x)$ does not exist.
8. **Column - I**

(A) $\lim_{x \rightarrow 0} [\sin |x| - |x|] =$

(B) $\lim_{x \rightarrow 0} \left[\frac{x}{[x]} \right] =$

(C) $\lim_{x \rightarrow \frac{1}{2}} \left[x \left[\frac{1}{x} \right] \right] =$

(D) $\lim_{x \rightarrow -1} \left[\frac{[x]}{x} \right] =$

(\because where $[.]$ denotes greatest integer function)

Column - II

(p) 0

(q) 1

(r) Does not exist

(s) -1

Answers Key

1. (C) 2. (A) 3. 1005 4. (D)

5. (i) $\frac{1}{\sqrt{2}}$ (ii) 6

6. $\left(0, \frac{1}{\sqrt{27}}\right] \cup \left[\frac{1}{3}, \sqrt{243}\right] \cup [27, \infty)$

7. $a = 1, 2, 3.$

8. (A) \rightarrow (s), (B) \rightarrow (r), (C) \rightarrow (p), (D) \rightarrow (q)

